A STUDY OF HEAT TRANSFER AND GAS DYNAMICS IN VESSELS CONNECTED THROUGH A COMPOUND MAIN

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An algorithm is developed for solving on a digital computer the differential equations which describe the heat transfer and the gas dynamics in a system consisting of two vessels connected through a compound main.

In engineering applications it is often necessary to analyze the heat transfer and the gas dynamics in vessels connected through a compound main. Such problems may be encountered, for instance, when gas flows from one container to another, when pipelines in cryogenic apparatus are chilled, or when the working process in single-stage gas-engine regenerators of refrigerators is studied.

The processes occurring during a one-dimensional unsteady and nonisothermal gas flow through pipes of uniform or variable cross section, at sudden channel expansions or contractions, and during emptying or filling of vessels have already been considered in [1, 2], where solutions for special cases have also been obtained. Numerical methods of solving the problem of unsteady gas flow through pipes were shown in [3, 4], but there the boundary conditions were defined in terms of certain time-dependent parameters of the gas stream. Gas transport between connected vessels was considered in [5-8], but the effect of the main on the gas flow characteristics was either not taken into account or was only roughly estimated. A system of differential and difference equations was set up in [9] for a refrigerator operating on the Stirling cycle, but the method of their solution was not given.

In this study the authors consider the problem of determining unsteady one-dimensional temperature, pressure, and velocity fields under conditions of gas transport between connected vessels, taking into account the heat transfer between the gas and the channel wall or a heat storing insert. The mathematics of this problem is somewhat extraordinary, because the conventional boundary conditions for the system of partial equations describing the gas flow and the heat transfer in the main must be replaced by the constraints of its coupling to the vessels. Since new unknown quantities are introduced here, namely the gas parameters in the vessels, it becomes necessary to use the equations of heat and energy balance in the vessels – ordinary differential equations. An algorithm for solving such a problem numerically is the object of this study.

The calculation procedure is shown schematically in Fig. 1: two vessels V_1 and V_2 of variable volume and arbitrary shape connected through a main. The vessels can also be connected to external reservoirs where the gas parameters are assumed constant.

The problem is solved under the following assumptions: a) heat transfer by conduction through the gas and through the pipe walls as well as radiation through the gas are negligible in comparison with the convective heat transfer; b) the thermal conductivity of the porous insert material filling a segment of the main is equal to zero in the direction of gas flow and infinitely high in the transverse direction; c) the walls of the main and of the vessels satisfy the condition $\mathrm{Bi} = \alpha\delta_c/\lambda_c \ll 1$ so that the temperature drop across the wall thickness becomes negligible; d) the working gas is an ideal gas and its specific heat is constant; e) the volume forces in the gas stream are negligibly small.

The processes in the vessels can be described by a system of ordinary differential equations with the thermodynamic parameters averaged over the vessel volume [1]. The flow rate in the subsequent

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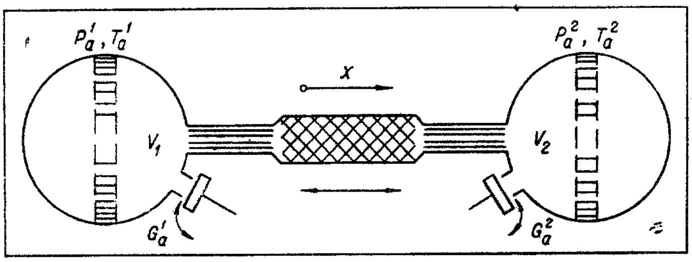

Fig. 1. Generalized calculation schematic.

equations will be considered positive during filling and negative during emptying (z = 0 when n = 1, z = y when n = 2).

The energy equation for the gas is

$$\frac{d}{dt}(m_n e_n) = -P_n \frac{dV_n}{dt} + \Sigma G_{en} H_{en} - F_{cn}\alpha_n (T_n - T_{cn}) + q_n. \tag{1}$$

The heat balance equation for a wall is

$$\rho_{cn}\delta_{cn} \frac{de_{cn}}{dt} = \alpha_n (T_n - T_{cn}) + \alpha_{an} (T_{an} - T_{cn}) + q_{cn}. \tag{2}$$

The change of mass of gas in a vessel is

$$\frac{dm_n}{dt} = \Sigma G_{en}. \tag{3}$$

The equation of state is

$$P_n V_n = m_n R T_n. \tag{4}$$

The change of vessel volume, as a function of time, is

$$V_n = V_n(t). \tag{5}$$

The change of valve cross section, as a function of time, is

$$f_n = f_n(t). \tag{6}$$

The energy exchange with an external reservoir $G_{en}H_{en}$ is determined from the condition of quasi-steady gas flow from the reservoir to the filling vessel. In order to determine the flow rate G_{en}, we use expressions for a one-dimensional isentropic flow [10].

The heat transfer and the hydrodynamic processes in the main are described by the following system of one-dimensional partial differential equations in variables (mean values over the cross section) [1, 2]:

the flow equation

$$\frac{\partial}{\partial t} F\rho U + \frac{\partial}{\partial x}\beta_1 F\rho U^2 = -F\frac{\partial P}{\partial x} - \frac{\zeta}{2d_h} F\rho U|U|; \tag{7}$$

the continuity equation

$$\frac{\partial}{\partial t} F\rho + \frac{\partial}{\partial x}(F\rho U) = 0; \tag{8}$$

the energy equation for the gas

$$\frac{\partial}{\partial t}\left[F\rho\left(i + \beta_1\frac{U^2}{2} - \frac{P}{\rho}\right)\right] + \frac{\partial}{\partial x}\left[F\rho U\left(i + \alpha_1\frac{U^2}{2}\right)\right] = -\frac{4\alpha}{d_h} F\left(T - T_c + \frac{U^2}{2c_p}\right); \tag{9}$$

the heat balance equation for the wall

$$\rho_c \frac{\partial e_c}{\partial t} = S\left(T - T_c + \frac{U^2}{2c_p}\right) + W(T_a - T_c) + q_c, \tag{10}$$

where $S = \alpha/\delta_c$, $W = \alpha_a/\delta_c$ for the wall and $S = (4\alpha/d_h)[p/(1-p)]$, $W = 0$ for the insert;

the equation of state

$$P = \rho RT. \tag{11}$$

This system of equations must be supplemented by the temperature characteristics of those thermophysical properties which are subject to appreciable variations over the analyzed temperature range, because such variations may have an appreciable effect on the processes (as has been shown in [11]). Coefficients α_1 and β_1 are further assumed both equal to unity.

As the initial conditions we stipulate any arbitrary distributions of gas temperature, gas pressure, gas velocity, gas mass, vessel wall temperature, and main wall temperature:

$$\begin{gathered} P_n(0) = P_n^0;\quad T_n(0) = T_n^0;\quad T_{cn}(0) = T_{cn}^0;\quad m_n(0) = m_n^0; \\ V_n(0) = V_n^0;\quad P(0,\ x) = P_0(x);\quad T(0,\ x) = T_0(x); \\ T_c(0,\ x) = T_{c0}(x);\quad U(0,\ x) = U_0(x). \end{gathered} \tag{12}$$

The system of equations (7)-(11) is hyperbolic and comprises four families of characteristics:

$$dx/dt = U;\qquad dx/dt = U \pm a;\qquad dx/dt = 0.$$

The trend of these characteristics calls for two boundary conditions at the entrance to the main and one boundary condition at the exit from the main [12]. Such a stipulation of boundary conditions is not feasible in this case, but equivalent constraints are available here: the coupling of the main to each vessel, with the number of constraints at the entrance to and at the exit from the main necessarily corresponding to the number of required boundary conditions. Thus, for subsonic gas flow, at the entrance we use two constraints ($U < a$, a denoting the velocity of sound in the gas stream [1]):

$$\begin{gathered} P_n = P_0 + \zeta_0\rho_0 U_0^2 \text{ — conservation of momentum;} \\ \frac{2k}{k-1} RT_n = \frac{2k}{k-1} RT_0 + U_0^2 \text{ — conservation of energy.} \end{gathered} \tag{13}$$

As the coupling constraint at the exit we use the conservation of momentum (the gas pressure in the jet entering a vessel and the gas pressure in the vessel are not very different; this is not true of the temperatures and, therefore, using the conservation of energy as the coupling constraint serves no purpose here):

$$P_n = P_y + \frac{F_y}{F_n}\zeta_y\rho_y U_y^2 \text{ — conservation of momentum.} \tag{14}$$

If the exit is replaced by a blind wall, then a constraint is provided which represents a boundary condition here ($U = 0$).

The resulting system of differential equations and constraints (1)-(14), which describes the heat transfer and the gas dynamic characteristics of a unidirectional flow of gas between connected vessels, is solved by the finite-differences method. The original equations, after a few transformations, are approximated by a system of difference equations (first-order implicit scheme [12]):

$$\frac{T_n^{i+1} - T_n^i}{\tau} = -(k-1)T_n^{i+1}A_1 + \frac{1}{m_n^i}\Sigma G_{en}^{i+1}[k(T_{en}^*)^i - T_n^i] - A_2\frac{V_n^i}{m_n^i}T_n^{i+1} + A_2\frac{V_n^i}{m_n^i}T_{cn}^i + \frac{q_n}{c_v m_n^i}, \tag{15}$$

$$\frac{P_n^{i+1} - P_n^i}{\tau} = -kP_n^{i+1}A_1 + \frac{R}{V_n^i}\Sigma G_{en}^{i+1}k(T_{en}^*)^i - A_2\frac{V_n^i}{m_n^i}P_n^{i+1} + A_2 RT_{cn}^i + \frac{q_n R}{c_v V_n^i}; \tag{16}$$

$$\frac{m_n^{i+1} - m_n^i}{\tau} = \Sigma G_{en}^{i+1}\beta, \tag{17}$$

where

$$A_1 = (V_n^{i+1}/V_n^i - 1)/\tau;\quad A_2 = \alpha_n\frac{4}{d_n c_v};$$

$$G_{\text{en}}^{i+1}=\begin{cases} G_a^{i+1} \text{ — energy exchange with reservoir.}\\ U_z^{i+1}\rho_z^i F_z \text{ — energy exchange with main.}\end{cases}$$

When a vessel is filled

$$\beta=1;\quad (T_{\text{en}}^*)^i=\begin{cases} T_a \text{ — energy exchange with reservoir.}\\ T_z^i+\dfrac{(U_z^{i+1})^2}{2c_p} \text{ — energy exchange with main.}\end{cases}$$

When gas is discharging from a vessel

$$(T_{\text{en}}^*)^i=T_n^i;\quad \beta=\begin{cases} m_n^{i+1}/m_n^i \text{ — discharge into reservoir}\\ 1 \text{ — discharge into main.}\end{cases}$$

The gas flow rate G_a^{i+1} is determined according to [10]:

$$\frac{T_{cn}^{i+1}-T_{cn}^i}{\tau}=\frac{1}{\rho_{cn}\delta_{cn}c_{cn}}\left[\alpha_n(T_n^{i+1}-T_{cn}^{i+1})+\alpha_{an}(T_{an}^i-T_{cn}^{i+1})+q_{cn}\right], \tag{18}$$

$$\frac{U_j^{i+1}-U_j^i}{\tau}+U_j^i\frac{U_j^{i+1}-U_{j-1}^{i+1}}{h}+R\frac{T_j^i}{P_j^i}\cdot\frac{P_j^{i+1}-P_{j-1}^{i+1}}{h}=-\frac{\zeta}{2d_h}U_j^{i+1}|U_j^i|, \tag{19}$$

$$\frac{P_j^{i+1}-P_j^i}{\tau}+U_j^i\frac{P_j^{i+1}-P_{j-1}^{i+1}}{h}-\frac{P_j^i}{T_j^i}\left(\frac{T_j^{i+1}-T_j^i}{\tau}+U_j^i\frac{T_j^{i+1}-T_{j-1}^{i+1}}{h}\right)$$
$$+P_j^i\frac{U_j^{i+1}-U_{j-1}^{i+1}}{h}=-\frac{B_1B_2}{h}P_j^i, \tag{20}$$

where $B_1=U_j^{i+1}$ for $B_2>0$ and $B_1=U_{j-1}^{i+1}$ for $B_2<0$, $B_2=1-F_{j-1}/F_j$;

$$\frac{T_j^{i+1}-T_j^i}{\tau}+U_j^i\frac{T_j^{i+1}-T_{j-1}^{i+1}}{h}-\frac{k-1}{k}\cdot\frac{T_j^i}{P_j^i}\left(\frac{P_j^{i+1}-P_j^i}{\tau}+U_j^i\frac{P_j^{i+1}-P_{j-1}^{i+1}}{h}\right)=-\frac{4\alpha}{d_h}\cdot\frac{k-1}{k}$$
$$\times\frac{T_j^i}{P_j^i}\left[T_j^{i+1}-T_{cj}^i+(U_j^i)^2/2\cdot c_p\right]+\frac{\zeta}{2d_h}\cdot\frac{1}{c_p}(U_j^i)^2|U_j^i|. \tag{21}$$

$$\frac{T_{cj}^{i+1}-T_{cj}^i}{\tau}=\frac{1}{\rho_c c_c}\left\{S\left[T_j^{i+1}-T_{cj}^{i+1}+\frac{(U_j^i)^2}{2c_p}\right]+W(T_a^i-T_{cj}^{i+1})+q_c\right\}. \tag{22}$$

The coupling constraints are

$$\left.\begin{aligned} P_n^{i+1}&=P_0^{i+1}+\zeta_0\rho_0^i(U_0^{i+1})^2\\ \frac{2k}{k-1}RT_n^{i+1}&=\frac{2k}{k-1}RT_0^{i+1}+(U_0^{i+1})^2\end{aligned}\right\}, \tag{23}$$

$$P_n^{i+1}=P_y^{i+1}+\frac{F_y}{F_n}\zeta_y\rho_y^i(U_y^{i+1})^2. \tag{24}$$

The system of difference equations (19)-(21) is solved by the method of orthogonal elimination [13]. For this purpose, Eqs. (19)-(21) are written as

$$A_j^iX_{j-1}^{i+1}+B_j^iX_j^{i+1}=C_j^i, \tag{25}$$

where A_j^i, B_j^i are square matrices of coefficients for a certain time interval; $X=\begin{pmatrix}U\\P\\T\end{pmatrix}$ is a vector whose components are the sought gasthermodynamic parameters in the main.

The coupling constraints are expressed as

$$\left.\begin{array}{l} L_0^i X_0^{i+1} = \varphi_0^i \\ N_y^i X_y^{i+1} = \psi_y^i \end{array}\right\}, \tag{26}$$

where L_0^i, N_y^i are rectangular matrices, 2×3 and 1×3, respectively, formed from Eqs. (23) and (24) by eliminating P_n^{i+1} and T_n^{i+1} with the aid of Eqs. (15)-(16) and a subsequent linearization with respect to velocity ($U^{i+1} = U^i + \Delta U^i$). The computation of gas stream parameters in the main is then organized as shown in [13]. After U_j^{i+1}, P_j^{i+1}, and T_j^{i+1} for the main have been determined from known discharge velocities U_0^{i+1} and U_y^{i+1}, we find the gasthermodynamic parameters in the vessels from Eqs. (15)-(17).

In the case of emptying a main which is closed at one end, another relation (in addition to $U = 0$) is required according to (26). As this extra relation we use the characteristic equation (corresponding to the $dx/dt = U$ characteristic), which for $U = 0$ becomes

$$\frac{T_0^{i+1} - T_0^i}{\tau} - \frac{k-1}{k} \cdot \frac{T_0^i}{P_0^i} \cdot \frac{P_0^{i+1} - P_0^i}{\tau} = -\frac{4\alpha}{d_h} \cdot \frac{k-1}{k} \cdot \frac{T_0^i}{P_0^i} (T_0^{i+1} - T_{c0}^i). \tag{27}$$

The temperature of the vessel walls T_{cn}^{i+1} and T_{cj}^{i+1} is found from Eqs. (18) and (22), respectively. Thus, all gasthermodynamic parameters in the vessels and in the connecting main have been determined for the $(i + 1)$-th time interval. A time interval should be approximately equal to 0.005-0.01 of the total process time (for calculating the discharge from vessel to the reservoir this time interval should be 0.001-0.002 of the total process time); the space interval should be approximately equal to 0.03-0.05 of the total main length.

This procedure was used with a model M-222 digital computer for calculating the transient mode in a single-stage gas-engine regenerator of a refrigerator operating on the MacMahon cycle (the feasibility of computing the steady-state mode in this and in some other refrigeration engines was considered in [14]) and for calculating the chill in the pipeline of cryogenic apparatus based on a gas cycle. The heat transfer and the drag coefficients were determined from respective empirical relations as in [15, 16]. The thermophysical properties of the gas and of the heat-transfer surfaces in these calculations were picked according to [17] and assumed constant.

For calculating the chill in the engine (cylinder diameter 0.04 m, piston stroke 0.04 m, speed 250 rpm, working gas helium, refrigeration output 5 W), V_1 was a volume equal to the total volume of all hot gas-supply channels from the reservoir to the cold compartment, V_2 was the displacer, and the main was the regenerator connecting both. As the apparatus runs into the operating mode, the insert temperature at the cold end of the regenerator varies as shown in Fig. 2 (curves 1, 2, 3), dropping almost linearly from 300 to 150°C. The duration of the transient is not the same at all sections of the regenerator: stabilization in the hotter sections of the insert proceeds faster. The steady-state temperature distribution in the insert along the regenerator (Fig. 2, curves 4, 5, 6), with the thermodynamic parameters in the vessels assumed invariable (variations of the gas temperature and the gas pressure in the displacer during the operating cycle are shown in Fig. 3), departs from the linear distribution based on constant boundary conditions [11]. The pressure losses in the regenerator occur essentially during filling and emptying the system, and they may be as high as $2 \cdot 10^5$ N/m^2 (Fig. 2, curve 7).

For calculating the chill in the pipeline, vessel V_1 is the gas pad containing liquefied nitrogen and receiving gas at a constant flow rate of 0.1 kg/sec, vessel V_2 is the gas container or the surrounding space. A pipeline (diameter 0.2 m, length 100 m, wall thickness 0.005 m, thermal influx 5 W/m^2, material: stainless steel) serves as the connecting main between vessels V_1 and V_2. Variations in both the gas and the wall temperature at various points along the pipeline during chilling are shown in Fig. 4. The distributions of both the gas and the wall temperature along the chilled pipeline may be considered approximately linear, with the underrecuperation almost uniform and equal to 5°C throughout the length.

This algorithm for solving the system of differential equations which describe steady-state heat transfer and gas dynamic processes in vessels connected through a compound main makes it feasible to calculate the variation of parameters in engineering devices and to incorporate these variations into the computation scheme.

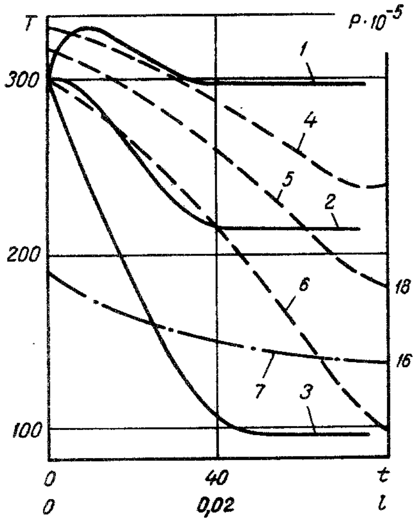

Fig. 2

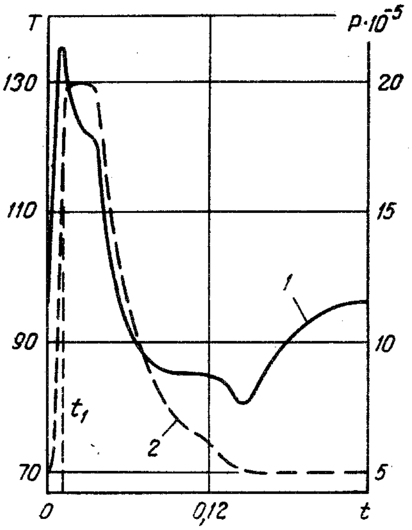

Fig. 3

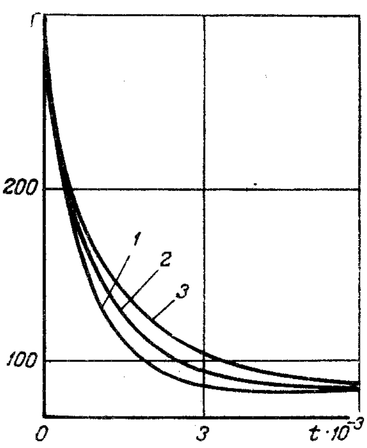

Fig. 4

Fig. 2. Variation in the insert temperature T (°K) at various regenerator sections during a transient [1) $l = 0$; 2) 0.02; 3) 0.04 m], along the regenerator [4) t = 10; 5) 20; 6) 75 sec]; 7) variation of the gas pressure P (N/m^2) along the regenerator (at instant of time t_1).

Fig. 3. Variation in the gas temperature (1) and pressure (2) during the working cycle under steady-state conditions: temperature T (°K); time t (sec); pressure P (N/m^2).

Fig. 4. Variation in the wall temperature (°K) of the chilled pipeline [1) $l = 0$; 2) 50; 3) 100 m].

NOTATION

T is the gas temperature;
T_c is the wall temperature;
P is the pressure;
U is the velocity;
e is the internal energy;
HI is the total enthalpy;
hi is the enthalpy;
t is the time;
x is the length;
m is the mass of gas in a vessel;
ρ is the density of gas;
ρ_c is the density of wall material;
G is the gas flow rate;
V is the volume;
α is the heat-transfer coefficient;
ξ is the drag coefficient;
q is the thermal flux;
d is the diameter of cylinder;
d_h is the hydraulic diameter;
c_p is the specific heat of gas at constant pressure;
c_v is the specific heat of gas at constant volume;
c_c is the specific heat of wall material;
δ is the wall thickness;
f is the area of valve cross section;
F is the active cross section of gas stream;
F_{cn} is the heat-transfer surface;
R is the gas constant;
p is the porosity;
k is the adiabatic exponent;
l is the length of main;

t_r is the duration of process;
τ is the time interval in computation;
h is the space interval in computation;
r is the number of time computation points;
y is the number of space computation points;
α_1, β_1 are the coefficients of velocity and density nonuniformity across a section of the main.

Subscripts

0 denotes the initial state;
n denotes the vessel number;
z denotes the coupling point between main and vessel;
a denotes the ambient medium;
w denotes the wall;
i denotes the time point;
j denotes the space point.

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